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21MATCS41

Fourth Semester B.E. Degree Examination, Dec.2023/Jan.2024
Mathematical Foundations for Computing, Probability & Statistics

Time: 3 hrs.   Max. Marks: 100

Note: 1. *Answer any FIVE full questions, choosing ONE full question from each module.*
2. *Use of Data tables are permitted.*

Module-1

1   a. Define Tautology. Verify that
$[P \rightarrow (Q \rightarrow r)] \rightarrow [(P \rightarrow Q) \rightarrow (P \rightarrow r)]$ is a tautology.   **(06 Marks)**

   b. Using the laws of logic, prove the following logical equivalence:
$[(\neg P \vee Q) \wedge \{P \vee (P \wedge Q)\}] \Leftrightarrow P \wedge Q$ **(07 Marks)**

   c. Write down the following proposition in symbolic form and find the negation,
" For all integers n, if 'n' is not divisible by 2 then 'n' is odd".   **(07 Marks)**

OR

2   a. Given that P, Q, R are propositions having truth values 0, 0, 1 respectively. Find the truth value of the following propositions :
   (i) $P \rightarrow (Q \wedge r)$.
   (ii) $(P \vee Q) \vee r$
   (iii) $(P \wedge Q) \rightarrow r$ **(06 Marks)**

   b. Write the following argument in symbolic form and then establish the validity :
If A gets the supervisor's position and works hard, then he will get a raise.
If he gets a raise, then he will buy a car.
He has not purchased a car.
Therefore he did not get the supervisor's position or he did not work hard.   **(07 Marks)**

   c. Write (i) A direct proof and (ii) An indirect proof
"If n is an odd integer, then (n+9) is an even integer".   **(07 Marks)**

Module-2

3   a. If $A = \{a_1, a_2, a_3, a_4\}$ and $B = \{b_1, b_2, b_3\}$. Find the following :
   (i)   Number of function from A to B as well as B to A.
   (ii)   Number of onto functions and one-one function from A to B.   **(06 Marks)**

   b. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by aRb if and only if 'a' is a multiple of b. Represent R as a set of ordered pairs. Draw the digraph and matrix representation of R.   **(07 Marks)**

   c. Prove the following for the graph G = (V, E) :
   (i) $\sum_{v \in V} \deg(v) = 2|E|$.
   (ii)   The number of vertices of odd degree must be even.   **(07 Marks)**

OR

4 a. Let $f : R \rightarrow R$ be defined by,
$$f(x) = \begin{cases} 3x - 5 & \text{if } x > 0 \\ -3x + 1 & \text{if } x \leq 0 \end{cases}.$$

Find $f(1)$, $f\left(-\frac{5}{3}\right)$, $f^{-1}(3)$, $f^{-1}([-5, 5])$. **(06 Marks)**

b. Let $A = \{1, 2, 3, 4\}$, let R be a relation on A defined by xRy iff x/y and y = 2x. Write down the following :
   (i)   R as a relation of set of ordered pairs.
   (ii)   Digraph of R.
   (iii) Indegree and Outdegree of the vertices in the graph. **(07 Marks)**

c. Define Graph isomorphism. Determine whether the following graphs are isomorphic or not.
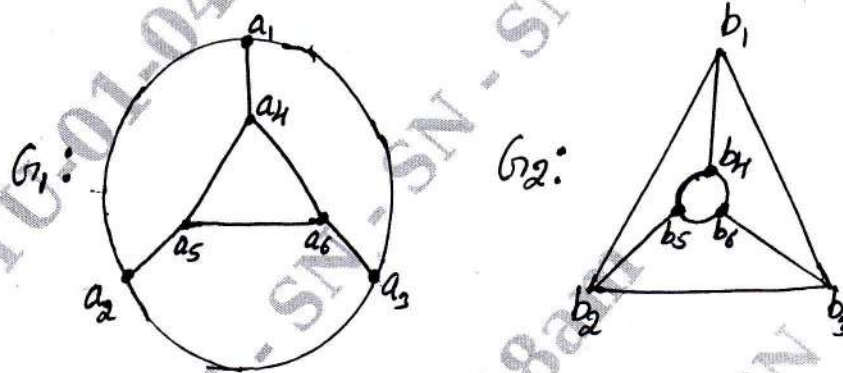

Fig. Q4 (c) **(07 Marks)**

Module-3

5 a. The following are the marks of 8 students in statistics and mathematics.

Marks in statistics	25	43	27	35	54	61	37	45
Marks in Mathematics	35	47	20	37	63	54	28	40

Calculate the rank correlation coefficient. **(06 Marks)**

b. Fit a best fitting curve in the form $y = ax^b$ for the following data :

x	350	400	500	600
y	61	26	7	26

(07 Marks)

c. Fit a second degree parabola, $y = ax^2 + bx + c$ in the least square method. For the following data and hence estimate y at x = 6.

x	1	2	3	4	5
y	10	12	13	16	19

(07 Marks)

OR

6 a. Fit a straight line in the form $y = ax + b$ by the least square sense for the following data :

x	5	10	15	20	25
y	16	19	23	26	30

(06 Marks)

b. Fit a best fitting parabola $y = ax^2 + bx + c$ for the following data:

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

(07 Marks)

c. The following table gives the heights of fathers (x) and sons (y) ;

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

Calculate the coefficient of correlation and lines of regression. (07 Marks)

Module-4

7 a. A random variable X has the following probability function :

x	0	1	2	3	4	5	6
P(x)	K	3K	5K	7K	9K	11K	13K

Find K, and evaluate ; $P(x \geq 5)$ and $P(3 < x \leq 6)$. (06 Marks)

b. Find the mean and standard deviation of Poisson distribution. (07 Marks)

c. In a test on electric bulbs, it was found that the life of a particular brand was distributed normally with an average life of 2000 hours and S.D of 60 hours, if a firm purchase 2500 bulbs find the number of bulbs that are likely to last for,
    (i)   More than 2100 hours.
    (ii)   Less than 1950 hours.
    (iii)   Between 1900 to 2100 hours. (07 Marks)

OR

8 a. Find the value of K such that the following distribution represents a finite probability distribution. Hence find its mean and standard deviation.

x	-3	-2	-1	0	1	2	3
P(x)	K	2K	3K	4K	3K	2K	K

(06 Marks)

b. Find the mean and standard deviation of binomial distribution. (07 Marks)

c. If the probability of a bad reaction from a certain injection is 0.001 determine the chance that out of 2000 individuals more than two will get a bad rejection. (07 Marks)

Module-5

9 a. Determine (i) Marginal distribution (ii) Covariance between the discrete random variable X and Y of the joint probability distribution :

X \ Y	3	4	5
2	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
5	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
7	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

(06 Marks)

b. A machinist is making engine parts with axle diameter of 0.7 inch. A random sample of 10 parts shows mean diameter 0.742 inch with a S.D of 0.04 inch. On the basis of this sample would you say that the work is inferior? (07 Marks)

c. A die is thrown 264 times and the number appearing on the fact (x) follows the following frequency distribution ;

x	1	2	3	4	5	6
f	40	32	28	58	54	60

Calculate the value of X^2. (07 Marks)

OR

10 a. Explain the terms : (i) Null hypothesis (ii) Confidence intervals
   (iii) Type I and Type II errors. **(06 Marks)**

   b. Four coins are tossed 100 times and the following results were obtained :

No. of heads	0	1	2	3	4
Frequency	5	29	36	25	5

   Fit a binomial distribution for the data and test the goodness of fit $\left(\chi^2_{0.05} = 9.49 \text{ for } 4 \text{ d.f}\right)$
   (07 Marks)

   c. The nine items of a sample have the following values : 45, 47, 50, 52, 48, 47, 49, 49, 53, 51.
   Does the mean of these differ significantly from the assumed mean of 47.5? **(07 Marks)**

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